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String theory and holography: open strings as closed strings

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Abstract

String theory provides strong evidence that quantum gravity with certain asymptopia is described by a conventional—nongravitational—gauge theory living on the boundary. In this short summary of the plenary lecture we will introduce this correspondence together with a summary of recent progress in the construction of the quantum gravity description of nonlocal operators in gauge theory such as Wilson loop operators.

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1. Introduction

String theory originated in the 1960s from an attempt to understand the strong interactions. In that period, hadronic particles with ever increasing spin were produced in particle accelerators. This proliferation of hadronic states led the theorist of the time to suspect that not all these particles could be fundamental, as at the time there was no known consistent theory of fundamental particles of high spin, a problem that still persists to this day.

A deceivingly simple and beautiful idea was put forward to address this embarrassment of riches. The idea was to think of the various hadronic particles of mass m and spin s as different oscillatory modes of an extended one-dimensional object, a fundamental string. In particular, the mesons were interpreted as excitations of an open string.

This simple model led to some phenomenological successes. It explained in an elegant way the experimentally observed 'Regge trajectories', where the maximal spin s_{max} of a hadron as a function of the mass m of the hadron fitted a linear trajectory

$$m^2 \simeq \frac{s_{\text{max}}}{\alpha'} + \text{const}, \tag{1.1}$$

where α' is the so-called Regge slope, which phenomenologically $\alpha' \simeq (\text{GeV})^{-2}$. In the string model, the phenomenological formula (1.1) follows immediately from the kinematics of rotating strings. The tension of the string T is inversely proportional to the Regge slope α'.

Better experimental data together with improved theoretical understanding of the consequences of string theory posed a serious challenge for string theory as the theory of the strong interactions. The correct theory—quantum chromodynamics—was proposed shortly after and immediately superseded string theory as the correct theory of the hadronic world. Better data at higher energies demonstrated that string theory exhibited a much softer behavior than that seen at colliders. QCD, on the other hand, spectacularly accounted for this high-energy regime.

On the theoretical side a deeper understanding of the consistency conditions also led to serious discrepancies with the hadronic world. Unitarity of the *S*-matrix predicted that string theory made sense only in higher dimensions, above four spacetime dimensions. Also string theory predicted the presence of a massless spin two particle, which is not present in the hadronic world.

So the frontal approach of string theory for the hadronic world failed. Nevertheless, it was realized that string theory was an ideal candidate for a theory of quantum gravity [1, 2]. The annoying massless spin two particle was shown to couple in precisely the same way as the graviton does, which is the quantum for the gravitational force. So string theory went from a theory of the strong interactions to a unified theory, as it naturally contained gravity and other gauge forces. The closed strings describe gravity and the open strings the other gauge interactions.

In a twist of irony, it is has now been understood that a certain string theory might describe quantum chromodynamics, the correct theory of the strong interactions. The connection is both subtle and deep. The strings no longer propagate in flat space as it was originally envisioned. They propagate in a curved geometry and motion in the extra dimensions has the gauge theory interpretation as renormalization group flow. This relation between string theory as quantum theory of gravity and a non-gravitational gauge theory is dubbed as a holographic correspondence [4, 3].

Currently, we do not yet have the correct string theory description of quantum chromodynamics. But the string theory description of other interesting four-dimensional gauge theories have been proposed. Remarkable progress has been made in encoding the physics of gravity in terms of gauge theory.

2. Holographic correspondence

The holographic correspondence states that string theory—i.e. quantum gravity—with specified boundary conditions can be alternatively described by a non-gravitational gauge theory. This connection is very deep as it posits that quantum gravity can be described by a set of degrees of freedom that are not gravitational. Understanding how to encode the degrees of freedom of the gauge theory in the gravitational description may shed light on some of the most profound puzzles of quantum gravity, such as the physics near the big bang and when black holes evaporate.

This equivalence or duality in quantum gravity is called holographic because there is a precise sense by which the gauge theory lives at the boundary of the spacetime where we are studying quantum gravity. The physics in the bulk of spacetime—which we will refer as bulk—is encoded in terms of a gauge theory that lives at the boundary.

In this talk, we will concentrate on the simplest case where we study quantum gravity with asymptotically AdS_5 boundary conditions [5–7], as this is the case where the holographic correspondence in understood in most detail. AdS_5 or five-dimensional Anti-de Sitter space is the maximally symmetric space with a negative cosmological constant. It solves Einstein's equations

$$R_{ij} = -\frac{4}{L^2} g_{ij}, \tag{2.1}$$

where L is the radius of curvature of AdS_5. Geometrically, AdS_5 can be described by a hyperboloid in $R^{2,4}$ via

$$-X_{-1}^2 - X_0^2 + X_1^2 + \cdots + X_4^2 = -L^2. \tag{2.2}$$

This representation makes manifest the $SO(2,4)$ isometry of AdS_5. Physically, AdS_5 behaves like a cavity as light reaches the boundary in finite time and the physics of the bulk is therefore sensitive to the boundary conditions imposed at the boundary, a fact that is crucial for the holographic correspondence.

The holographic four-dimensional dual gauge theory lives at the boundary of AdS_5. In this correspondence the vacuum state in the bulk—global AdS_5—is identified with the vacuum state of the gauge theory. Since AdS_5 has an $SO(2,4)$ isometry, this means that the four-dimensional gauge theory is $SO(2,4)$ invariant, namely it is invariant under the four-dimensional conformal group.

The correspondence now implies that for every excited state in the gauge theory there is a corresponding state of string theory which is asymptotic to AdS_5. Since a generic excitation in the gauge theory breaks conformal invariance, the geometry in the bulk[1] needs to be AdS_5 only asymptotically. An early example of the identification between a gauge theory state and a bulk geometry is the identification [8] of the thermal state of the gauge theory at temperature T with a black hole in AdS_5 with the Hawking temperature given by T, which is described by the AdS–Schwarzwild metric.

The physical information in the gauge theory is captured by the correlation function of gauge-invariant observables [5–7]. The simplest observables to consider first are the correlators of local gauge-invariant operators. A given correlation function in the gauge theory is computed from the bulk point of view by evaluating the bulk path integral with a precise choice of boundary conditions which encode the information about the operators involved in the correlator.

An important challenge in this program is to identify the asymptotically AdS_5 geometries that describe all the gauge-invariant operators in the gauge theory. Chief among these operators are Wilson loop operators—characterized by curves in spacetime—which can be used to give a manifestly gauge-invariant description of the gauge theory. One can also construct gauge theory operators which are characterized by a surface in spacetime. Apart from their relevance for holography, these operators may also serve as new-order parameters for new phases of gauge theories.

3. Holographic Wilson loops

A necessary step in describing string theory in terms of a holographic dual gauge theory is to be able to map all gauge-invariant operators of the field theory in string theory, as all physical information is captured by gauge-invariant observables.

Gauge theories can be formulated in terms of a non-Abelian vector potential or alternatively in terms of gauge-invariant Wilson loop variables. The formulation in terms of non-Abelian connections makes locality manifest while it has the disadvantage that the vector potential transforms inhomogeneously under gauge transformation and is therefore not a physical observable. The formulation in terms of Wilson loop variables makes gauge-invariance manifest at the expense of a lack of locality. The Wilson loop variables, being

[1] Here we are glossing over the fact that, in general, the bulk description of a gauge theory state might not have a semiclassical geometric description, but may only have a very stringy non-geometric description.

non-local, appear to be the natural set of variables in which the bulk string theory formulation should be written to make holography manifest. It is therefore interesting to consider the string theory realization of Wilson loop operators[2].

We find that a Wilson loop operator can have multiple, completely equivalent descriptions in the bulk[3] [11] (see also [12–14]). Which description is used depends on which of the descriptions is most semiclassical, so it is computationally most tractable . We find that there is a description which might be called the 'probe' description [11, 13] and then there is another one described in terms of exact solutions of supergravity [12, 15–17] that are asymptotically AdS_5.

3.1. Wilson loops in AdS

A Wilson loop operator is labeled by a curve C in spacetime and by a representation R of the gauge group G. The data that characterizes a Wilson loop, the curve C and the representation R, label the properties of the external particle that is used to probe the theory. The curve C is identified with the worldline of the particle propagating in spacetime while the representation R corresponds to the charge carried by the particle.

Apart from the curve C, the other piece of data entering into the definition of a Wilson loop operator is the choice of representation R of the gauge group G. For gauge group $U(N)$, the irreducible representations are conveniently summarized by a Young tableau $R = (n_1, n_2, \ldots, n_N)$, where n_i is the number of boxes in the ith row of the tableau and $n_1 \geqslant n_2 \geqslant \cdots \geqslant n_N \geqslant 0$. The corresponding Young diagram is given by

1	2	·	·	·	·	n_1
1	2	·	·	·	n_2	
1	2	·	·	·	n_3	
·	·	·	·			
1	2	·	n_N			

.

The main goal is to identify all half-BPS Wilson loop operators of $\mathcal{N} = 4$ SYM in the dual asymptotically AdS gravitational description.

The information about the curve C is encoded in the bulk by considering states in string theory that end on the boundary of AdS_5 along the curve C. If we consider an excitation of string theory in AdS_5 that ends on a curve C on the boundary that has the effect of introducing the appropriate source to insert a Wilson loop operator defined on C, given by

$$W_R(C) = \mathrm{Tr}_R P \exp\left(\mathrm{i}\int_C \mathrm{d}s(A_\mu \dot{x}^\mu + \phi_I \dot{y}^I)\right), \tag{3.1}$$

where C labels the curve $(x^\mu(s), y^I(s))$ and P denotes path ordering along the curve C.

For the problem at hand there are three basic excitations that can end on the AdS_5 boundary on a curve, they can be either a fundamental string, a $D5$-brane or a $D3$-brane. It was shown early on that the fundamental string describes a Wilson loop in the fundamental representation of the gauge group, where the Young tableau just has one box.

Physically, one expects that by stacking multiple strings together that one can obtain Wilson loops in higher-dimensional representations of the gauge group. It turns out that this intuition is indeed correct. More precisely, one can show that k coincident strings may be described in terms of a single $D5_k$-brane or a single $D3_k$-brane. Physically what happens is

[2] This has been done for Wilson loops in the fundamental representation by authors of [9, 10].

[3] We will concentrate in the dictionary for the most symmetric case corresponding to maximally supersymmetric gauge theory in four dimensions.

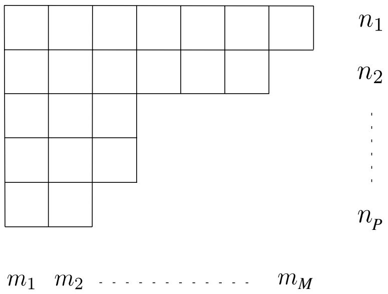

Figure 1. A Young tableau. For $U(N)$ $P \leqslant N$ and M is arbitrary.

that the coincident fundamental strings blow up into a D-brane with dissolved electric flux, which encodes the information about the fundamental string charge. In analogy with the pointlike case, we denote the $D5_k$-brane as the giant Wilson loop and the $D3_k$-brane as the dual giant Wilson loop.

For this particular problem we can explicitly show that a $D5_k$-brane corresponds to a Wilson loop in the kth antisymmetric representation while the $D3_k$-brane corresponds to a Wilson loop in the kth symmetric representation of the gauge group.

The strategy to show this is to integrate out the degrees of freedom introduced by adding the $D5_k$- or $D3_k$-brane and see that their net effect is to insert the Wilson loop operator (3.1) in the so-mentioned representation. Physically, adding a $D5_k$-brane adds one-dimensional fermions on the loop C while adding a $D3_k$-brane adds one-dimensional bosons. The path integral over the fermions/bosons can be explicitly performed and result with the insertion of the desired Wilson loop.

One is able to show that a Wilson loop labeled by the Young tableau can be described in terms of M $D5$-branes or alternatively in terms of P $D3$-branes in AdS_5.

There is yet another bulk description for a half-BPS Wilson loop. One may try to take the gravitational backreaction of the collection of M $D5$-branes or P $D3$-branes in AdS_5. Since D-brane source the gravitational field one must find solutions where the D-branes are replaced by the gravitational backreaction they produce. The backgrounds that are produced are asymptotically AdS_5 and describe an excitation above AdS_5 due to the Wilson loop operator insertion. The boundary conditions that determine the bulk gravitational background can be read from the Young tableau in figure 1.

In summary, we have given a complete bulk description of all half-BPS Wilson loop operators. They can be explicitly shown to be described either by D-branes or asymptotically AdS_5 geometries[4]. We can therefore successfully identify important gauge-invariant operators of the gauge theory on the boundary in terms of bulk entities.

The evidence is by now very compelling. Gauge theories in the boundary of spacetime seem to capture the physics of quantum gravity in the bulk of spacetime. It remains to be seen whether powerful enough techniques can be developed to construct and solve the string theory description of quantum chromodynamics. When that happens, string theory will then finally describe the theory of the strong interactions.

[4] In the plenary talk, we also described how this identification of Wilson loops with D-branes and bubbling Calabi–Yau manifolds can be also be shown [18, 19] in the topological string theory context and also for higher-dimensional surface operators [20, 21].

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